

## Math 436 (Spring 2020) - Homework 2

1. **Chapter 2:** 13, 14
2. Prove the following statements:
  - (a) The set  $\mathbb{Z}$  of all integers is closed but not open in  $\mathbb{R}$ .
  - (b) The set  $\mathbb{Q}$  of all rational numbers is neither closed nor open in  $\mathbb{R}$ .
3. Let  $X$  and  $Y$  be two topological spaces. If  $f: X \rightarrow Y$  is a constant map (that is,  $f$  maps all points in  $X$  to the same point in  $Y$ ), then  $f$  is continuous.
4. Let  $X$  be a discrete topological space (that is, every subset of  $X$  is open). Given any topological space  $Y$ , show that any map  $f: X \rightarrow Y$  is continuous.
5. Let  $X$  be the subspace  $[0, 1] \cup [3, 4]$  of  $\mathbb{R}$ . Show that the map  $f: X \rightarrow \mathbb{R}$  defined by

$$f(x) = \begin{cases} 0 & \text{if } x \in [0, 1] \\ 1 & \text{if } x \in [3, 4] \end{cases}$$

is continuous.